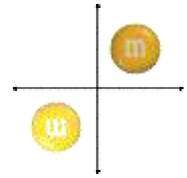
Flips, Spins, and Slides…Oh My!



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Logistics: This lesson is intended for students in Grades 6 – 8 in an introductory pre-algebra or geometry course as a hands-on investigation into several transformational geometry concepts. This specific lesson focuses on students creating "rules" for different transformations based upon observed patterns in the coordinates of an individual point.

Materials:

per student:

1 – copy of Student Pages

per pair of students:

1 – 11 x 17" coordinate grid board (laminated) (available from Learning Resources www.learningresources.com)
10 – mini M&Ms baking pieces or pony beads
1 – Mira plastic reflecting device (available EAI Education www.eaieducation.com)
1 – Safe-T angle ruler (available from www.schoolmart.com)
3 – pieces linguine pasta (uncooked)
1 – dry erase marker
1 – roll transparent tape
Glue dots (optional)

Time: This lesson can be completed in two 50-minute class periods.

Objectives/Standards:

- Develop "rules" for isometric transformations of coordinates.
  CCSS.Math.Content.8.G.A.1 **Verify experimentally the properties of rotations, reflections, and translations**
- Plot coordinates of original (pre-image) and transformed (image) points for figures on a coordinate plane.
  CCSS.Math.Content.6.NS.C.6.b **Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.**
- CCSS.Math.Content.6.NS.C.6.c **Find and position pairs of integers and other rational numbers on a coordinate plane.**
- CCSS.Math.Practice.MP2 **Reason abstractly and quantitatively**
- CCSS.Math.Practice.MP4 **Model with mathematics**
- Use Mira reflective mirrors and Angle Rulers
  CCSS.Math.Practice.MP5 **Use appropriate tools strategically**
- CCSS.Math.Practice.MP7 **Look for and make use of structure**

NOTES

Introduction: Transformational geometry involves movement of a figure through translations (slides), reflections (flips), and rotations (spins) on a coordinate plane. These movements, when applied to two-dimensional figures drawn on the coordinate plane, allow students to test for congruency between two or more figures. Translations, reflections, and rotations preserve the size and shape of the figure. In addition to these transformations, enlarging or reducing the size of an object through dilation allows students to describe similarity between figures by identifying congruent angles and proportional side lengths.

Students will be introduced to translations, reflections and rotations by using a coordinate grid and several M&M baking pieces to investigate the movement of a point on the coordinate plane. They will then construct general “rules” for the movement of a point for each of these different transformations by analyzing patterns in the coordinates and will test their rules using a three-sided figure created using M&M baking pieces and linguine.

Advanced Preparation: Be sure to have coordinate grids available for students that are large enough to accommodate either M&M baking pieces or pony beads as points. Label the x-axis from -10, 10 and the y-axis from -10, 10 and encourage students to choose points from within this window.

Activity:

This activity will be broken into three parts. The first part will introduce students to translations on the coordinate plane. The next part introduces reflections followed by rotations. It is intended for the class to debrief their findings after every part of the activity is completed.

Part I: Translations

Students should be placed in partner teams for this activity.

Provide each team with one 11” x 17” laminated coordinate grid board, approximately 10 mini M&M baking pieces (approximately 5 each of two different colors), a ruler or straight edge, and a dry erase marker.

1” square ruled easel pads and sticker dots can be purchased from Office Depot

If necessary, review with students the placement of points on a four-quadrant coordinate grid. If available, mark a sheet of 1” grid paper with an x-axis and y-axis and ask students to place sticker dots at coordinates that are called out.

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Pass out the student pages and have partner teams read the background information in the gray box. Circulate throughout the room to verify that teams have correctly identified the four quadrants on the coordinate grid. Encourage teams to make careful observations as they work through *Part I* of the procedure. Allow adequate time for students to complete *Part I* and then debrief this part of the activity by asking the following questions:

- **How did your team define a geometric translation?** [*A translation on the coordinate plane involves moving every point on the original, or pre-image, that same distance and in the same direction to produce the translated image.*]

- **What rule did your team discover for translating any point on a coordinate plane?** [*If a point (x, y) is translated h units horizontally and v units vertically, than* $(x, y) \rightarrow (x + h, y + v)$*where h is positive when moving to the right; negative to the left, and v is positive moving upward and negative moving downward.*]

  NOTE: Understand that as students are developing towards writing an abstract equation, that they may generate several variations of the above rule. This is an opportunity to look at all of the rules generated by the teams and discuss their equivalence (if any), how to test their validity, and how to refine them for clarity.

Part II: Reflections

Students should continue in their partner teams from *Part I*.

In addition to the materials students already have at their tables, provide each partner team with one Mira plastic reflecting mirror and one piece of linguine. Demonstrate the proper way to view images in the Mira by showing students how to place the beveled edge of the Mira at the bottom facing the figure that they will be reflecting. They will then look through the Mira to see the reflected image. Proper placement along the line of reflection is very important for accurate results.

Mira™ can be purchased through EAI Education

NOTE: If students do not have Miras, provide them with tracing paper in which they will mark the locations of the M&Ms and will then fold over the line of reflection (piece of linguine) to locate the reflected points.

Allow students adequate time to work through the procedure for *Part II*.

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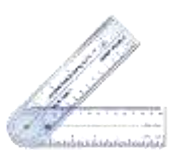
They will be investigating reflections over both the x-axis and the y-axis. Reflecting over other lines of reflection can be investigated as an extension activity. The third activity in this series (*TANSformations*) has students use lines of reflection other than the axes. Therefore, it would be interesting to have students describe a "procedure" using the linguine for reflecting points over any horizontal or vertical line. The extension further asks students to consider reflecting over the line $y = x$ or $y = -x$. Allow adequate time for students to complete *Part II* and then debrief this part of the activity by asking the following questions:

- **How did your team define a geometric reflection?** [*A reflection is a flip of a point or object over a line of reflection to produce a mirror image of the object.*]

- **What rule(s) did your team discover for reflecting a point on a coordinate plane over the axes?** [*If a point (x, y) is reflected over the y-axis (x = 0), the coordinates of the y-coordinate of the reflected point will be opposite $(x, y) \rightarrow (x, -y)$. However, if reflecting over the x-axis (y = 0), the x-coordinate will become opposite while the y-coordinate remains the same $(x, y) \rightarrow (-x, y)$.*]

Part III: Rotations

SAFE-T Angle Rulers can be purchased through Learning Resources



Students should continue working in their partner teams from *Part I*.

In addition to the materials students already have at their tables, provide each partner team with one angle ruler. Demonstrate to students how to use their new tool by showing them how the small brass circular area will be placed at the origin and the closed ruler will rotate so that the horizontal line on the ruler intersects a point of interest. Students may use their dry erase pens to highlight the horizontal lines on the angle ruler originating from the 0° location on the protractor portion of the angle ruler.

The students will place an M&M on the ruler at a point and will rotate the top piece of the ruler to the desired location while holding down the bottom piece of the ruler and keeping the center of rotation at the origin. To assist students, you may choose to provide transparent tape to "hold down" the bottom portion of the ruler and/or glue dots to place on the ruler so that the M&M will not slide as students rotate the angle ruler.

NOTE: If students do not have angle rulers, a substitute can be constructed using two strips of transparency film (with a line drawn down

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the center of both strips with a permanent marker) and a brass fastener (pushed through the left end of both strips along the horizontal line). Around the brass fastener, use a protractor and permanent marker to mark the locations of 0 degrees (horizontal to the right), 90 degrees, 180 degrees, and 270 degrees on the top transparency strip.

Allow students adequate time to work through the procedure for *Part III*. They will be investigating rotations around the origin at 90 degrees (CW or CCW), 180 degrees (CW or CCW), 270 degrees (CW or CCW), and 360 degrees (CW or CCW).

Debrief this part of the activity by asking the following questions:

- **What rules did your team discover about rotations about the origin for the angles of rotation specified?**

  90° CW (270° CCW): $(\boldsymbol{x}, \boldsymbol{y}) \rightarrow (\boldsymbol{y}, -\boldsymbol{x})$

  180° CW (180° CCW): $(x, y) \rightarrow (-x, -y)$

  270° CW (90° CCW): $(x, y) \rightarrow (-y, x)$

  360° CW (360° CCW): $(x, y) \rightarrow (x, y)$

- **What are the coordinates for triangle ABC after a rotation of 90° CCW about the origin?** [A' (10, -20), B' (20, -10, C' (5, -8)

  after a rotation of 180° CCW about the origin?
  [A'' (20, 10), B'' (10, 20), C'' (8, 5)]

Working in groups of two to four, allow students time to discuss the answers to the activity debrief questions, and then debrief with the entire class.

Debrief questions:

- ***What information is necessary in order to translate a figure?***
  [A translation needs both direction and distance to move horizontally and/or vertically.]

- ***What information is necessary in order to reflect a figure?***
  [A reflection needs a line of reflection.]

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- ***What information is necessary in order to rotate a figure?***
  [Rotations need a center of rotation, degree of rotation, and a direction of rotation.]

- ***For the transformations investigated in this activity (translations, rotations, and reflections), what characteristic(s) of a figure change?*** [the orientation and location of the figure change]

  What characteristic(s) of the figure stay the same?
  [the size and the shape of the figure after undergoing one or more of these "rigid" transformations is the same]

- ***Describe another transformation that would be similar to reflecting a figure over the x-axis and then over the y-axis.***
  [A rotation of 180° about the origin]

CONCLUSION:
This lesson is intended to introduce students to isometric transformations that preserve the size and shape of the various figures. The enduring understanding that we would like students to take away is that two figures can be considered congruent if one can be "constructed" through performing one or more isometric transformations on the other figure. This will then lead into a discussion about how a non-rigid transformation (dilation) can produce similarity in figures through enlargements or reductions.

EXTENSIONS:
For an online interactive look at transformations, students can navigate to http://www.shodor.org/interactivate/activities/Transmographer/ or to https://illuminations.nctm.org/Activity.aspx?id=6475 (requires JAVA).

An extension to this activity is also provided for students to investigate reflecting over other lines other than the axes and for rotating about points other than the origin. You may also choose to introduce "double" transformations by asking students to perform a series of transformations such as a translation followed by a reflection. Students can hypothesize whether the order of performing the transformations matters.

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RESOURCES:

Congruent Triangles (and other shapes). (n.d.). Retrieved February 01, 2016, from http://www.helpingwithmath.com/by_subject/geometry/geo_congruent_8g2.htm

Reviewing Transformations. (n.d.). Retrieved January 29, 2016, from http://www.regentsprep.org/regents/math/algtrig/atp9/reviewTranformations.htm

Transformations in the Coordinate Plane. (n.d.). Retrieved January 29, 2016, from http://www.virtualnerd.com/middle-math/integers-coordinate-plane/transformations

Additional Websites:

http://www.harcourtschool.com/activity/icy_slides_flips_turns/

http://www.hoodamath.com/mobile/games/transformationgolf.html

https://www.khanacademy.org/math/geometry/congruence/transformations-congruence/e/exploring-rigid-transformations-and-congruence

NOTES

Transformations

In this series of activities, you will be learning about different geometric transformations on the coordinate plane. If you have ever seen the movie *Transformers*, why do you think they were given their name?
Many professionals including video game designers, architects, engineers, artists, graphic designers, medical imaging technicians, and movers use transformations in their daily work. You will investigate several different types of transformations, and will look at patterns to develop definitions and rules for various types of transformations.

Problem: *Develop rules to describe different transformations of a point on a coordinate plane.*

Materials per group:

- 1 – 11 x 17” coordinate grid board (laminated)
- 10 – mini M&Ms baking pieces or pony beads
- 1 – Mira plastic reflecting device
- 1 – Angle ruler
- 1 – straight edge or ruler
- 1 – piece uncooked linguine
- 1 – dry erase marker

Procedure:

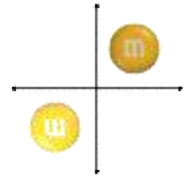
Part I: Translations

1.) With a partner, you will use an 11 x 17” coordinate grid board, several M&M baking pieces, a ruler, and a dry erase marker for this part of the activity. Using the dry erase marker, label the quadrants I, II, III, and IV. Have your teacher check your work before you continue.

2.) Now, take one M&M baking piece and place it anywhere on the coordinate grid where there are intersecting grid lines. For this activity, you will be tracking coordinates, so to make it easier to see patterns, choose points that have integer values for coordinates. Record the coordinates of the M&M: (,   )

3.) Slide the M&M baking piece to the right. Record the new coordinates of the M&M: (,   ). How many units did you slide the M&M to the right? ___________
How did that impact the value of the coordinates?

____



4.) Repeat Step 3 for slides to the left, up, and down, placing a new M&M for each different slide. Record your data below:

Original M&M Coordinates	Direction of Slide	Number of Units Point "Slid"	New M&M Coordinates	How are the new coordinates different from the original?
	Right			
	Left			
	Up			
	Down			

5.) Using the data in the table above, what pattern(s) do you observe when points are "moved" horizontally?

How about when points are "moved" vertically?

6.) Now, place another M&M piece in any quadrant on the coordinate grid (record the ORIGINAL color here: ______________) and place a different colored M&M diagonally from the first M&M (record the NEW color here: ______________).

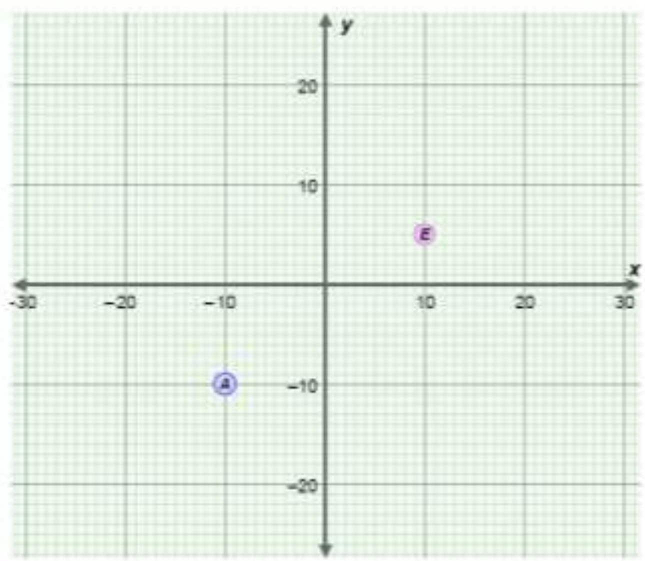
ORIGINAL M&M Coordinates: (,) NEW M&M Coordinates: (,)

7.) Describe the movement of the M&M point from the ORIGINAL location to the NEW location using words such as x-coordinate, y-coordinate, horizontal, and vertical:

8.) Can you write a mathematical expression for your NEW coordinates by using the ORIGINAL coordinates? EXAMPLE: Original (-3, 6) New (-3 + 5, 6 – 7)

ORIGINAL: (_______, _______) NEW: (_______________, _______________) 9.) Now, write a "rule" for any point, (x, y), if it is translated h units horizontally and v units vertically:

10.) Use your rule to describe the movement of Point A to its ending location at Point E:



11.) Now, use the dry erase pen to mark three different points in the same quadrant. Connect the points using a straight-edge (ruler). What shape is this? ______________
Place M&Ms at the coordinates of the **vertices** (endpoints) of the figure you drew. Slide each M&M point _____ units horizontally and ____ units vertically. Draw the new figure using the location of the M&Ms at the coordinates of the vertices.

12.) Compare the original and the new figures.

13.) You have been exploring a type of transformation known as a **TRANSLATION**. In your own words, define a translation in the space below:

Part II: Reflections

1.) With a partner, you will use an 11 x 17” coordinate grid board (erase figures from previous activity), several M&M baking pieces, linguine, a Mira, and a dry erase marker for this part of the activity.

2.) Investigate the Mira. Notice that there is one edge of the Mira that is beveled. Always place the Mira with the beveled edge at the bottom facing the object of “interest.”

3.) Place the piece of linguine along the $\boldsymbol{y}$-axis. The equation of the vertical line formed by the piece of linguine is $x = 0$. This is how we can identify the location of the vertical line. Place three M&Ms on the piece of linguine and record their coordinates below.

(,) (,) (,)

What do you notice about the $\boldsymbol{x}$-coordinates?

How about the $\boldsymbol{y}$-coordinates?

If you moved the linguine two units to the right, what might be the equation of that vertical line? ______________

Place the linguine back on the $\boldsymbol{y}$-axis ($x = 0$)

4.) Now, take one M&M baking piece and place it at a point in the first quadrant. Record the coordinates of your M&M (,).

Place the Mira along the piece of linguine on the $\boldsymbol{y}$-axis so that the beveled edge faces the M&M. When you look through the Mira, you should see an image on the “other side” of the Mira. Take another M&M and place in the location of the image.

Record the coordinates of the M&M image (,)

What do you notice about the coordinates?

____

5.) Now, remove the M&Ms. Place M&Ms one at a time in different locations along the coordinate plane. Record the coordinates of the Mira image. This image is a **reflection** of the original point. There is a table on the following page for your data.

6.) The linguine should still be along the y-axis. However, you may slide the Mira up and down the linguine as needed. Record your data in the table below.

Original M&M Coordinates	Reflected M&M Coordinates	How are the coordinates of the M&M image reflected over the y-axis different from the original?

7.) Now, write a “rule” for any point, (x, y), if it is reflected over the y-axis:

8.) Remove all M&Ms from the coordinate grid and place the linguine along the x-axis. Place three M&Ms on the piece of linguine and record their coordinates below.

(,   ) (,   ) (,   )

What do you notice about the x-coordinates?

How about the y-coordinates?

What might be the equation of the horizontal line that is the x-axis? ______________

9.) Take one M&M baking piece and place it at a point in the first quadrant.

Record the coordinates of your M&M (,   ).

10.) Place the Mira along the piece of linguine on the x-axis so that the beveled edge faces the M&M. When you look through the Mira, you should see an image on the “other side” of the Mira. Take another M&M and place on the location of the image.

Record the coordinates of the M&M image (,)

What do you notice about the coordinates?

____

11.) Remove the M&Ms. Place M&Ms one at a time in different locations along the coordinate plane. Record the coordinates of the reflected Mira image.

The linguine should still be along the x-axis. However, you may slide the Mira back and forth as needed. Record your data in the table on the next page.

Original M&M Coordinates	Reflected M&M Coordinates	How are the coordinates of the M&M image reflected over the x-axis different from the original?

12.) Now, write a “rule” for any point, (x, y), if it is reflected over the x-axis:

13.) Now, use the dry erase pen to mark three different points in the same quadrant. Connect the points using a straight-edge (ruler) to make a triangle. Place M&Ms at the coordinates of the **vertices** (endpoints) of the triangle you drew.

Record the M&M coordinates: A (,) B (,) C (,) PREDICT: Using your rule for reflection over the y-axis, what would the coordinates of the vertices of the triangle be for the reflected image?

A' (,) B' (,) C' (,)

What do the letters A, B, and C refer to in this example?

What do the letters A', B', and C' refer to in this example?

14.) Use the Mira and additional M&Ms to reflect the triangle over the y-axis. Compare your predicted coordinates for the reflected image to the actual coordinates for the reflected image.

How is the reflected triangle different from a triangle that is translated?

15.) Now, repeat Steps 12 and 13 to reflect your triangle over the x-axis. Record your predicted and actual coordinates for the reflected image below.

PREDICTED coordinates of the vertices of reflected image:

A'(,) B'(,) C' (,)

ACTUAL coordinates of vertices of the reflected image using Mira:

A'(,) B'(,) C' (,)

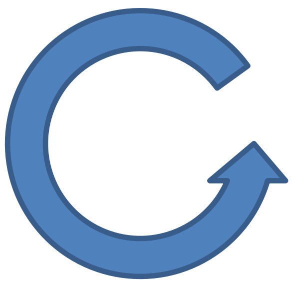
16.) You have been exploring a type of transformation known as a **REFLECTION**. In your own words, define a reflection in the space below:

Part III: Rotations

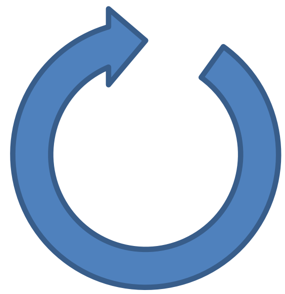
1.) With a partner, you will use an 11 x 17" coordinate grid board (erase figures from previous activity), several M&M baking pieces, an angle ruler, and a dry erase marker for this part of the activity.

2.) When an object spins, or rotates, we can refer to the **direction** of rotation using the terms **counterclockwise** (CCW) and **clockwise** (CW). Trace with your fingers around each circle in the direction of the arrow to get a feel for the direction of rotation:

Counterclockwise (CCW)



Clockwise (CW)



3.) The amount of rotation is known as the degree of **rotation**. How many degrees are there in a circle? ________

Therefore, how many degrees are in one rotation? _________

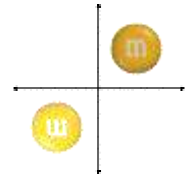
Look at your angle ruler. Align both the top and bottom of the ruler so that the horizontal lines are at 360 degrees (you may use your dry erase marker to highlight the long horizontal lines from 360° to the end of the ruler), which is the same as 0°. Why?

Now, rotate the top piece of the ruler until it aligns with 90°. What direction did you rotate? ___________________________

How many degrees would you have to rotate *in the opposite direction* (from 0°) to get to the same location? _______________________

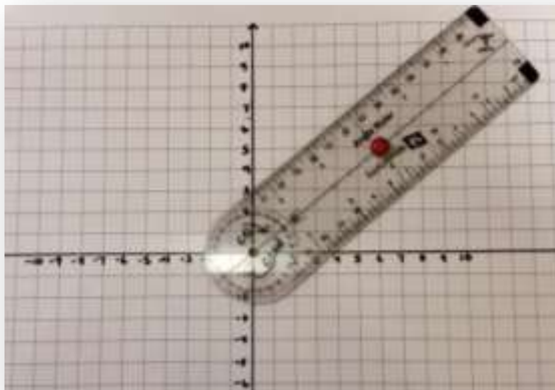
Using the angle ruler, answer the questions below:

90° CW = __________° CCW
180° CW = __________°CCW
270° CW = __________°CCW



4.) Align both pieces of the angle ruler so that they are at 0°. Place the small metal center of rotation on the angle ruler at the origin (0, 0). Spin the entire angle ruler into the first quadrant and place an M&M at a coordinate on the top of the angle ruler along the center line as shown below:

Your first point can be placed at (6, 5)



If provided, you may use transparent tape to hold down the bottom of the ruler and a glue dot to secure the M&M on top of the angle ruler.

5.) Holding the bottom part of the ruler down, carefully rotate the top of the ruler containing the M&M at (6, 5) 90° CW, 180° CW, 270° CW, and 360° CW about the origin and record the information in the table below.
Repeat for three additional points of your choice.

Original M&M Coordinates	Coordinates 90° CW (270° CCW)	Coordinates 180° CW (180° CCW)	Coordinates 270° CW (90° CCW)	Coordinates 360° CW (360° CCW)
(6, 5)				

6.) What patterns do you notice in the above table for the following rotations:

90° CW (270° CCW)	
180° CW (180° CCW)	
270° CW (90° CCW)	
360° CW (360° CCW)	

7.) Now, write a "rule" for any point, (x, y) rotated around the origin for the degrees listed below:

90° CW (270° CCW):

180° CW (180° CCW):

270° CW (90° CCW):

360° CW (360° CCW):

8.) Choose three points in one quadrant using the dry erase markers and connect the points using a ruler or straight edge.

Label the points of the triangle formed, **A**, **B**, and **C**. PREDICT the coordinates of the vertices of the rotated triangle after a rotation of **90° CCW** around the origin. Record your predictions below:

PREDICTED coordinates of the vertices of rotated image:

A' (,) B' (,) C' (,)

ACTUAL coordinates of the vertices of rotated image using M&Ms and angle ruler:

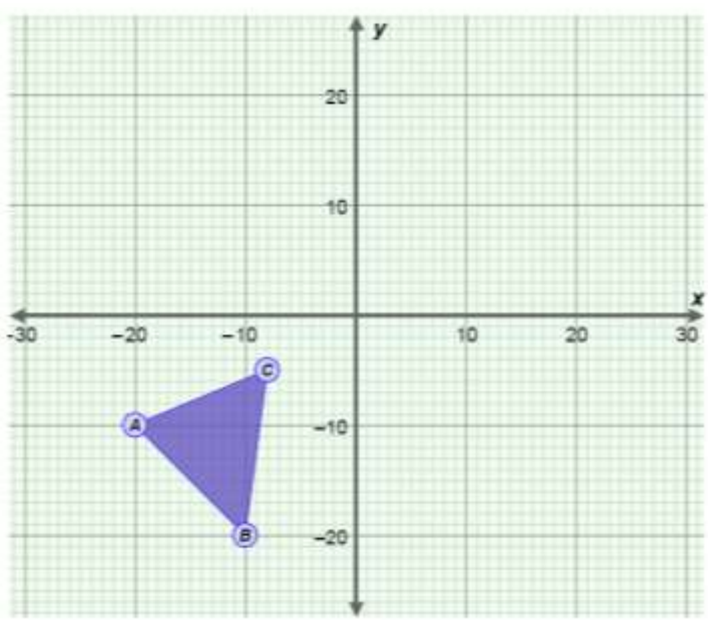
A' (,) B' (,) C' (,)

9.) On the grid to the right, draw the rotated image after a rotation of **90° CCW** around the origin.

A' (,)
B' (,)
C' (,)

180° CCW around the origin.

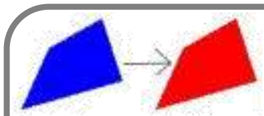
A'' (,)
B'' (,)
C'' (,)



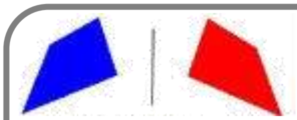
Debrief Questions

Complete the following questions with a partner. Then, share your answers as a class.

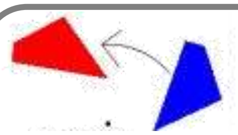
1.) What information is necessary in order to **translate** a figure?



2.) What information is necessary in order to **reflect** a figure?



3.) What information is necessary in order to **rotate** a figure?

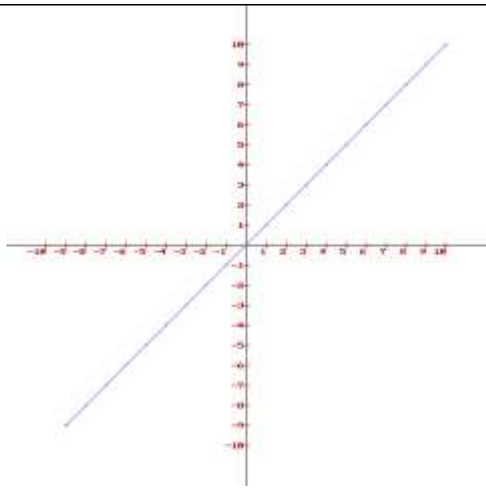


4.) What is true about all of the different transformations that were investigated in these activities – translations, reflections, and rotations?

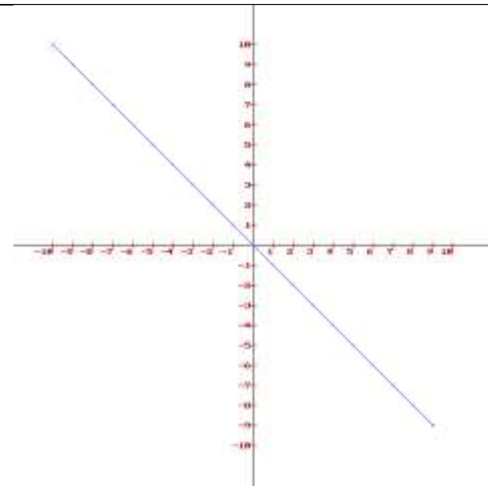
5.) CHALLENGE: How could you use translations, reflections, and rotations to show if two figures were exactly the same size and shape (congruent)?

EXTENSION:

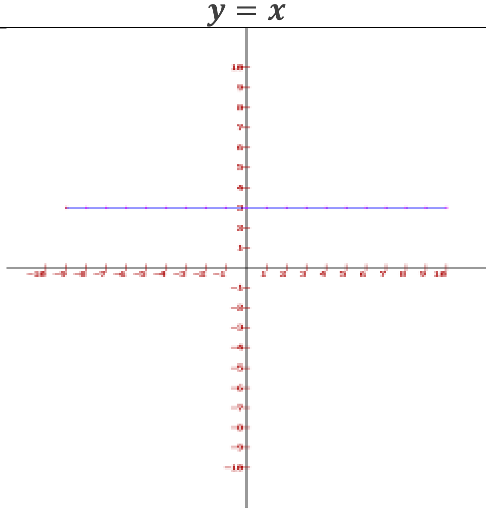
Using your coordinate grid board, M&Ms, linguine, and a dry erase marker, investigate the effect of reflecting points over the following lines:



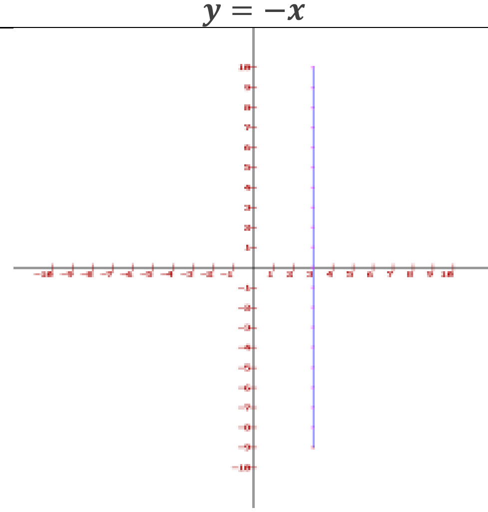
$y = x$



$y = -x$



$y = 3$



$x = 3$

What would be the effect of rotating a figure about a point other than the origin? Use your tools to investigate and record your results below: